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AMBIGUITY IN SOURCE FLUX OF COSMIC/ASTROPHYSICAL NEUTRINOS: EFFECTS OF BI-MAXIMAL MIXING AND QUANTUM-GRAVITY INDUCED DECOHERENCE *

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For high energy cosmic neutrinos Athar, Jeźabek, and Yasuda (AJY) have recently shown that the existing data on neutrino oscillations suggests that cosmic neutrino flux at the AGN/GRB source, $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 2 : 0$, oscillates to $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$. These results can be confirmed at AMANDA, Baikal, ANTARES and NESTOR, and other neutrino detectors with a good flavor resolution. Here, we re-derive the AJY result from quasi bi-maximal mixing, and show that observation of $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$ does not necessarily establish cosmic neutrino flux at the AGN/GRB source to be $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 2 : 0$. We also note that if the length scale for the quantum-gravity induced de-coherence for astrophysical neutrinos is of the order of a Mpc, then independent of the MNS matrix, the Liu-Hu-Ge (LHG) mechanism would lead to flux equalization for the cosmic/astrophysical neutrinos.

Keywords: Neutrino oscillations, bi-maximal mixing, LHG mechanism

1. Introduction

The solar neutrino anomaly, the LSND excess events, and the Super-Kamiokande data on atmospheric neutrinos, find their natural explanation in terms of oscillations of neutrinos from one flavor to another.[1,2,3] The only experiment so far that provides a direct evidence of oscillation from one flavor to another is the LSND experiment. However, the LSND result is still debated by the KARMEN collaboration.[4] It is expected to be settled by the dedicated Fermi Lab. experiments. Nevertheless, a strong tentative evidence for neutrino oscillations seems established. An indirect hint for neutrino oscillations resides in the problem of obtaining successful models of type-II supernova explosions. Neutrino oscillations, if confirmed to exist, can significantly aid these explosions:[5] in effect, neutrino oscillations, provide an indirect energy transport mechanism due to flavor-dependence of relevant neutrino cross sections.

In this *Letter* we shall neglect any possible CP violation in neutrino oscillations.

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We shall adopt the standard three-flavor neutrino scheme. In that framework one can accommodate any of the following two sets of data: (a) Data on the atmospheric neutrinos and solar neutrino anomaly, or (b) Data on atmospheric neutrinos and LSND excess events. In the quasi bi-maximal mixing, the angle θ, see Eq. (7) below, can accommodate either the LSND results or the solar anomaly, but not both.

In the context of this experimental setting, and the stated theoretical framework, this *Letter* establishes the abstracted result. The origin for the abstracted result lies in the observation that the observed L/E flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data strongly favors[7,8] a quasi bi-maximal mixing matrix (and in fact this is what drives the AJY result). Here we show that quasi bi-maximal mixing transforms $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : a : 2-a$ to $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$. Note, the latter flux neither carries an a dependence, nor is it affected by the angle θ. This robustness has the consequence that by studying the departures from the $F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \approx 1 : 1 : 1$ for the observed cosmic high energy flux one may be able to explore new and interesting sources/physics of high energy cosmic neutrinos. The data, however, may also be used to study unitarity-preserving deformations of bi-maximality.

In the next section, we summarize the AJY result under study. Section 3 shows the quasi bi-maximal mixing as the physical origin of flux equalization for AGN/GRBs, it then presents the theorem advertised in the *Abstract*, and it ends by introducing a deformed bi-maximal mixing and its affect on the flux equalization. Section 4 discusses the LHG mechanism in the context of an observation by Adler on quantum-gravity induced de-coherence effects. Section 5 is devoted to conclusion.

2. Brief review of AJY flux equalization

Without CP violation, the three-flavor neutrino oscillation framework carries five phenomenological parameters. These are the two mass-squared differences, Δm^2_{32} and Δm^2_{21}, and the three mixing angles:

$$U(\theta,\beta,\psi) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \quad 1 \quad & \quad\quad 2 \quad\quad & 3 \end{array} \\ \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta \\ -c_\theta s_\beta s_\psi - s_\theta c_\psi & c_\theta c_\psi - s_\theta s_\beta s_\psi & c_\beta s_\psi \\ -c_\theta s_\beta c_\psi + s_\theta s_\psi & -s_\theta s_\beta c_\psi - c_\theta s_\psi & c_\beta c_\psi \end{pmatrix} \end{array} \tag{1}$$

The columns of the mixing matrix U are numbered by the mass eigenstates, $\jmath = 1, 2, 3$, while the rows are enumerated by the flavors, $\ell = e, \mu, \tau$. Here, we have introduced the usual abbreviations: $c_x = \cos(x)$, and $s_x = \sin(x)$.

For a phenomenological study, the essential question is what are the parameters of the neutrino oscillations and what information may be extracted from them about particle physics, and astrophysical and cosmological processes/sources. New flavor-sensitive detectors with a collection area exceeding 1 km^2 shall provide us valuable information about the high-energy cosmic neutrino flux. This flux carries important

information about the conventional processes of AGNs and GRBs, but it may also serve as a probe of certain quantum gravity effects and explore possible violations of the equivalence principle.[9,10,11,12,13,14,15,16]

For high energy neutrinos, $E \gtrsim 10^6$ GeV, with sources in AGNs and GRBs, the source-detector distance far exceeds the kinematically induced oscillation lengths suggested by any of the solar, atmospheric, and the LSND data. Under these circumstances the AGN and GRB neutrino flux, F^S, at the source is roughly in the ratio:

$$F_e^S : F_\mu^S : F_\tau^S :: 1 : 2 : 0 \tag{2}$$

The oscillated flux, F_ℓ^D, measured at terrestrial detectors, becomes independent of the mass squared differences, and is given by:[6]

$$F_\ell^D = \sum_{\ell'} P_{\ell\ell'} F_{\ell'}^S \tag{3}$$

with

$$P_{\ell\ell'} = \sum_{\jmath} |U_{\ell\jmath}|^2 |U_{\ell'\jmath}|^2 \tag{4}$$

Using the solar, reactor, atmospheric, and the accelerator, neutrino data AJY have made a detailed numerical analysis. The result is [6]:[a]

$$\text{AJY's numerical analysis:} \qquad F_e^D : F_\mu^D : F_\tau^D :: 1 : 1 : 1 \tag{5}$$

Analytically,[17] AJY show that the above result follows from the data-dictated assumptions:

$$\begin{aligned} |U_{e3}|^2 &\ll 1, \\ \big||U_{\mu\jmath}|^2 - |U_{\tau\jmath}|^2\big| &\ll 1, \quad \jmath = 1,2,3. \end{aligned} \tag{6}$$

3. Quasi Bi-maximal origin of flux equalization and an ambiguity theorem

We now show that this result is in fact a direct consequence of the quasi bi-maximal mixing inferred from the L/E-flatness of the electron-like event ratio observed in the Super-Kamiokande data on atmospheric neutrinos. Then, in the next section, we show that the flux equalization is not a unique signature of the source flux given by Eq. (2).

It was argued in Refs. [7, 8] that the observed L/E-flatness of the electron-like event ratio in the Super-Kamiokande data on atmospheric neutrinos places severe *analytical* constraints on the mixing matrix. Without reference to the solar neutrino

[a] Also see, Ref. [17]

deficit, or the data on the LSND excess events, it was shown that these constraints yield a quasi bi-maximal mixing matrix.[b] This result is contained in Eq. (26) of Ref. [8], and reads:

$$U = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta/\sqrt{2} & c_\theta/\sqrt{2} & 1/\sqrt{2} \\ s_\theta/\sqrt{2} & -c_\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \tag{7}$$

The mixing matrix (7), when coupled with Eq. (4), yields:[c]

$$P = \begin{pmatrix} s_\theta^4 + c_\theta^4 & c_\theta^2 s_\theta^2 & c_\theta^2 s_\theta^2 \\ c_\theta^2 s_\theta^2 & \frac{1}{4}\left[1 + s_\theta^4 + c_\theta^4\right] & \frac{1}{4}\left[1 + s_\theta^4 + c_\theta^4\right] \\ c_\theta^2 s_\theta^2 & \frac{1}{4}\left[1 + s_\theta^4 + c_\theta^4\right] & \frac{1}{4}\left[1 + s_\theta^4 + c_\theta^4\right] \end{pmatrix} \tag{8}$$

Substituting the obtained P in Eq. (3) furnishes with the prediction:

$$\text{Quasi Bi-maximal mixing:} \qquad F_e^D : F_\mu^D : F_\tau^D :: 1 : 1 : 1 \tag{9}$$

This is precisely the result (5) which AJY obtained based on a detailed numerical analysis.[6] On the analytical side,[17] the AJY constraints (6) are manifestly satisfied by the quasi bi-maximal mixing matrix (7).

Clearly, the AGN/GRB related $\mathcal{F}^S$ satisfy this flux equalization criterion with $a = 2$. For supernovae explosions, $a \approx 1$. Once again, one obtains the flux equalization. The early results of Learned and Pakvasa[21], and Weiler *et al.*[22], are seen to follow as a special case associated with $\theta = 0$ and $a = 2$.

The result (9) is independent of the mixing angle θ − the angle relevant for the solar, or LSND, data (see Refs. [7, 8]). This implies that the high energy cosmic neutrino flux is robust in that it does not depend on the (vacuum) mixing angle obtained from the solar neutrino anomaly, or from the LSND data. This robustness can be exploited to systematically study other possible significant sources of neutrino flux, especially those which may arise from sources other than the decay of charged pions. The latter component of the neutrino flux may appear as a departure from the evenly proportioned flux of the three neutrino flavors discussed here. The departures may also serve as a probe of certain quantum gravity effects and possible violations of the equivalence principle.[9,10,11,12,13,14,15,16] However, we now emphasize that detecting a flux (9) does not necessarily imply the source flux to be (2).

In interpreting any deviations from the result (9), one must be careful to note the following ambiguity theorem. Let

$$\mathcal{F}^S \equiv F_e^S : F_\mu^S : F_\tau^S :: 1 : a : 2 - a, \quad 0 \le a \le 2 \tag{10}$$

[b] The quasi bi-maximal mixing reduces to the bi-maximal mixing for $\theta = \pi/4$. Apart from Refs. [7, 8], other early references on bi-maximal mixing are [18, 19, 20].

[c] An invertible quasi bi-maximal mixing matrix U, Eq. (7), necessarily yields a P matrix that is non-invertible. This mathematical observation shall underlie the physical content of the theorem to be presented below.

Then, under the already stated framework, the quasi bi-maximal mixing has the effect

$$\mathcal{F}^S \to \mathcal{F}^D \tag{11}$$

where

$$\mathcal{F}^D \equiv F_e^D : F_\mu^D : F_\tau^D :: 1 : 1 : 1 \tag{12}$$

The proof is straight forward.

From an aesthetic point of view, a view which is also consistent with the existing data, the quasi bi-maximal mixing is a strong candidate to emerge as the unitary matrix behind the neutrino oscillations. The widely discussed bi-maximal mixing,[7,8,18,19,20,23] as already noted, is a special case of the quasi bi-maximal mixing. In this special case one may introduce a unitarity-preserving deformation of the bi-maximality, and constrain it by the existing data as follows:

$$U' = \begin{pmatrix} c_\beta/\sqrt{2} & c_\beta/\sqrt{2} & s_\beta \\ -(1+s_\beta)/2 & (1-s_\beta)/2 & c_\beta/\sqrt{2} \\ (1-s_\beta)/2 & -(1+s_\beta)/2 & c_\beta/\sqrt{2} \end{pmatrix}, \quad \beta \ll 1 \tag{13}$$

This deformed bi-maximal mixing transforms $\mathcal{F}^S$ given by Eq. (10) into

$$\mathcal{F'}^D \equiv {F'}_e^D : {F'}_\mu^D : {F'}_\tau^D :: 1 : 1 + (a-1)s_\beta^2 : 1 + (1-a)s_\beta^2 \tag{14}$$

and carries an essentially unique signature for the deformation parameter β, and for the source flux parameter a (associated with the class of neutrino fluxes under consideration).

4. Flux equalization in LHG mechanism

In a pioneering paper,[24] LHG have argued that quantum-gravity induced EHNS de-coherence[25] can lead to a flux equalization for all flavors. However, the length scale at which such a de-coherence sets in has been a reason of some discussion.[26,27] The essential argument that Adler advances is, in essence, correct:[27] these are effects of the quantum-gravity induced de-coherence on relative phases, and not on the global phase, that are important. However, Lisi *et al.*[26] counter, in a "Note added" to their work, that, at present, one needs to take a purely phenomenological approach to study such effects. Without addressing the specific objection of Adler, Lisi *et al.* question the dimensional arguments of Adler. It is the apparent weakness of Adler's dimensional argument that saves in the end the suggestion of Lisi *et al.* – a suggestion we suspect may, in fact, be too optimistic for atmospheric neutrinos, but may carry viability for astrophysical neutrinos.

In the context of the argument advanced by Lisi *et al.* we note that dimensional arguments can indeed break down when not all the relevant dimensionless numbers are incorporated in one's calculation, or are not known. The latter observation finds

support in the circumstance that despite the standard "forty orders of magnitude argument" gravitationally-induced phases in the neutron interferometry were first observed.[28] The dimensionless numbers that were missed in the standard arguments were: (a) The mass of the Earth divided by mass of the neutron, a number that equals 3×10^{51}, and (b) The dimensions of the interferometer arm compared with the de Broglie wavelength of a thermal neutron (which yields another dimensionless number of about 10^{10}).[29]. Similar circumstance arises in detection of neutrinos (despite exceedingly small cross sections), and in the possibility of probing space-time fluctuations via gravity wave detectors.[34]

For atmospheric neutrinos, a subject of the study contained in Lisi *et al.*'s work, E is of the order of a few GeV, and L ranges from about 20 km to about 1.3×10^4 km. Compared with the atmospheric neutrinos, for astrophysical neutrinos one gains in L by a minimum factor of about 10^{15}, while for MeV range astrophysical neutrinos, E decreases by roughly three orders of magnitude. Combined, these numbers give a minimum net gain in L/E by a factor of about 10^{18}.[d]Furthermore, one cannot assume that the quantum-gravity induced de-coherence is independent of the relevant gravitational environment. Such a dimensionless number on the surface of the Earth is, $M_\oplus G/R_\oplus c^2$ (incorporating c explicitly, now) $= 7\times 10^{-10}$. Its counterpart on the surface of a neutron star is about 0.2. These are precisely these dimensionless numbers that yield the so called 20% gravitational-induced effect in the red-shift of flavor oscillation clocks in Ref. [5] (see Refs. [30, 31, 32, 33] for further discussion). In astrophysical environments with intense gravitational fields, by naive dimensional arguments, we may obtain another ten orders of magnitude in favor of the quantum-gravity induced de-coherence in neutrino oscillations.

The original arguments that suggested that the quantum-gravity induced de-coherence may be observable for atmospheric, or solar, neutrinos thus appears too optimistic. However, the relevance of quantum-gravity effects in neutrino oscillations in the cosmic/astrophysical contexts cannot be easily ruled out. If the length scale for the quantum-gravity induced de-coherence for astrophysical neutrinos is of the order of a Mpc (MegaParsec), then independent of the MNS matrix, the LHG effect would lead to the flux equalization for the astrophysical neutrinos.

5. Conclusion

The observed L/E flatness of the electron-like event ratio in the Super-Kamiokande atmospheric data strongly favors a quasi bi-maximal mixing for neutrino oscillations. This quasi bi-maximal mixing contains one unconstrained mixing angle, θ. The angle θ can either be used to accommodate the LSND excess events, or to explain to the long-standing solar neutrino anomaly. For high energy cosmic neutrinos, the Source-Detector distance far exceeds any of the relevant kinematically induced oscillations lengths. When this information is coupled with the Super-Kamiokande

[d]For higher E, L/E suffers a decrease from this value. However, VEP[16] and qVEP[9] effects increase [no co-relation is implied, and final effects are a complex web of different effects].

implied quasi bi-maximal mixing, characterized by the angle θ, we find that a whole range of neutrino fluxes, $\mathcal{F}^S$, defined in Eq. (10), and characterized by a, oscillate to equal fluxes of ν_e, ν_μ, and ν_τ. This result carries a remarkable robustness in its θ- and a- independence.

Observation of equal ν_e, ν_μ, and ν_τ fluxes from AGN/GRBs, and supernovae explosions, can be used to establish if they belong to flux class, $\mathcal{F}^S$, defined above. Deviations of these fluxes, $\mathcal{F}^D$, as observed in terrestrial detectors, from the ratio $1:1:1$ can become a robust measure of the departure of the source flux ratio from $1:a:2-a$. A detailed study of these departures carries the seeds to discover new physics, and to characterize cosmic neutrino sources. In particular, it is to be emphasized that a remains unmeasurable, if the mixing is quasi bi-maximal (of which, bi-maximal mixing is a special case). Furthermore, the angle θ, that, e.g., can be adjusted to resolve the solar neutrino anomaly, does not influence the expected flux equalization. Because the high-energy cosmic neutrino flux as detected in terrestrial laboratories is insensitive to the underlying mass-squared differences, measurements on the flavor spectrum of the high-energy cosmic neutrino flux can be used to probe a whole range of parameters associated with neutrino oscillations. Since each of these parameters – from those related to the deformed bi-maximal mixing, to those that characterize a whole range of quantum-gravity effects (including those violating the principle of equivalence) – is likely to yield a different signature, high-energy cosmic neutrinos provide a powerful probe into new physics.

The ambiguity that has been discussion of this *Letter* is further compounded if the length scale for the quantum-gravity induced de-coherence for astrophysical neutrinos turn out to be of the order of a Mpc (MegaParsec). Then, independent of the MNS matrix, the LHG effect would lead to the flux equalization for the astrophysical neutrinos.

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Note Added

After this work was accepted for publication, Keränen brought to my attention his work where he and his colleagues[35] investigate the problem studied in this *Letter* in the context of a two-doublet structure of the 4-neutrino mixing. In a future study, I intend to look at their scenario with a special emphasis on the L/E flatness of the e-like event ratio seen in the atmopsheric neutrino oscillation data, and see to what extent the ambiguity reported here survives in the extended frameworks of neutrino oscillations.

Learned and Pakvasa (LP)[21] – in a work predating the AJY result[17] – had also observed that a siginficant parameter space of the 3×3 neutrino oscillation

framework contains the flux equalization of different flavors. The LP study was based on a whole range of allowed values of the three mixing angles as deciphered from then-existing data on the solar and atmospheric neutrinos. Here, we have traced back the origin of the LP-AJY result to the bi-maximal mixing as implied by the L/E flatness of the e-like event ratio observed at Super Kamiokande for atmospheric neutrinos, and, in addition, have brought in a new element that a similar ambiguity in tracing back the source flux is introduced by certain quantum gravity effects.

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